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Non-BPS D-branes and enhanced symmetry in an asymmetric orbifold

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Abstract: In this paper properties of D-branes in a nine dimensional asymmetric orbifold are discussed, using a $(-1)^{F_L}\sigma_{1/2}$ projection, where F_L is the leftmoving space-time fermion number and $\sigma_{1/2}$ is a freely acting shift of order two. There are two types of non BPS D branes, which are stable at $R > 2$ and $R < 2$ respectively. At $R = 2$ there is a perturbative enhancement of gauge symmetry and the two types of branes are related by a global bulk symmetry transformation. At this point in the moduli space the associated boundary states are constructed using a free fermion representation of the theory. Some aspects of the enhancement of gauge symmetry in the S-dual type-$\tilde{\text{I}}$ theory are discussed.

Keywords: Superstrings and Heterotic Strings, M-Theory, D-branes, String Duality.

Contents

1. Introduction

It is well known that there are no points of enhanced gauge symmetry in the moduli space of toroidally compactified type-II superstring theories. Non-abelian gauge symmetries for type-II strings arise however in asymmetric orbifolds [1, 2] or in free fermionic constructions [3, 4, 5]. Possibly the simplest examples of such asymmetric orbifolds are given by type-II on $T^d/(-1)^{F_L}\sigma_{1/2}$ where F_L is the left-moving space-time fermion number and $\sigma_{1/2}$ is a shift of order two in the compactification lattice $\Gamma^{d,d}$. $(-1)^{F_L}$ projects out the supercharges coming from the leftmovers and the freely acting lattice shift $\sigma_{1/2}$ ensures that no supersymmetries will be reintroduced in the twisted sector. Hence such a theory has only sixteen supersymmetries and has $(4,0)$ space-time supersymmetry (where the numbers count right and leftmoving space-time supersymmetry charges in four dimensional units). Unlike in the case of the heterotic string there is one leftmoving worldsheet supersymmetry which remains unbroken together with four rightmoving ones as is necessary for $N = 4$ space-time supersymmetry [6].

D-branes [7] are extended objects on which open strings can end. They impose boundary conditions on worldsheet fields relating the left and rightmovers. In type-II and type-I theories D-branes can be BPS objects, i.e. they preserve half of the space-time supersymmetries. Since the asymmetric orbifolds discussed above do not have any leftmoving supersymmetries D-branes can not be BPS objects. Recently, there has been a great interest in nonsupersymmetric string theory and non BPS branes (see for example [8, 9, 10, 11] and references therein). In the following we will discuss some of the properties of non-BPS D-branes in the simplest example of the asymmetric orbifold defined above, which is type-IIA or IIB compactified on $S^1/(-1)^{F_L}\sigma_{1/2}$, where $\sigma_{1/2}$ is the shift $X \to X + \pi R$ and R is the radius of the circle.

Various aspects of D-branes in asymmetric orbifolds are discussed in the literature: D-branes in asymmetric orbifolds constructed using T-duality [12], using Sagnotti's method of open descendants [13], asymmetric orientifolds [14, 15] and asymmetric type-I theories [16].

2. Closed string spectrum

Firstly consider the compactification of type IIA/B on a circle of radius R. Identifying $X^9 \sim X^9 + 2\pi R$. The compactified momenta (p_L, p_R) lie on a lattice $\Gamma_{1,1}$ (setting $\alpha' = 2$)

$$p_L = \frac{m}{R} + \frac{nR}{2}, \qquad p_L = \frac{m}{R} - \frac{nR}{2}. \tag{2.1}$$

In this units the self-dual radius, i.e. the radius where the closed bosonic string has enhanced SU(2) $\times$ SU(2) gauge symmetry, is given by $R = \sqrt{2}$. Note that in a toroidal compactification of type II there are no points of enhanced non-abelian gauge symmetry, because the GSO projection removes from the spectrum all the massless states carrying only lattice momentum and winding which are necessary for symmetry enhancement. The one loop partition function of type IIA/B is given by

$$Z_{1,1} = \int \frac{d^2\tau}{\tau_2} \tau_2^{-9/2} \frac{1}{|\eta(\tau)|^{16}} (\chi_{8,v} - \chi_{8,c})(\bar{\chi}_{8,v} - \bar{\chi}_{8,s/c}) \sum_{p\in\Gamma_{1,1}} q^{\frac{1}{2}p_L^2} \bar{q}^{\frac{1}{2}p_R^2}. \tag{2.2}$$

Here $\chi_{8,o}, \chi_{8,v}, \chi_{8,s}, \chi_{8,c}$ are SO(8) characters corresponding to adjoint,vector, and the two conjugate spinor representations respectively.

String orbifolds are constructed from a closed string theory by projecting onto invariant states and adding twisted sectors [17]. The main consistency requirement of this procedure is modular invariance of the torus partition function [18]. At any radius R the asymmetric Z_2 orbifold can be defined by projecting with $g = (-1)^{F_L}\sigma_{1/2}$ where F_L is the leftmoving space-time fermion number and $\sigma_{1/2}$ is the shift by half the period of the circle $X \to X + \pi R$. Defining for lattice vectors $l \in \Gamma_{1,1}$, $l = (l_L, l_R)$ the inner product is given by $l \cdot l' = l_L \cdot l'_L - l_R \cdot l'_R$ The half period shift is then encoded in the lattice vector $v = (R/4, -R/4)$.

For definiteness we will consider the IIB compactification here. The twist in the time direction in the path integral is defined by

$$Z_{(g,1)} = \mathrm{tr}\left((-1)^{F_R} e^{2\pi i v\cdot p} q^{L_0} \bar{q}^{\bar{L}_0}\right). \tag{2.3}$$

Adding (2.2) and (2.3) gives the untwisted contribution to the orbifold partition function

$$
\begin{aligned}
Z_{untw} &= \frac{1}{2}(Z_{(1,1)} + Z_{(g,1)}) \\
&= \int \frac{d^2\tau}{\tau_2}\tau_2^{-9/2}\frac{1}{|\eta(\tau)|^{16}}(\chi_{8,v} - \chi_{8,c})\bar{\chi}_{8,v} \sum_{p\in\Gamma_{1,1}} (1 + e^{2\pi i p\cdot v}) q^{\frac{1}{2}p_L^2}\bar{q}^{\frac{1}{2}p_R^2} - \\
&\quad - \int \frac{d^2\tau}{\tau_2}\tau_2^{-9/2}\frac{1}{|\eta(\tau)|^{16}}(\chi_{8,v} - \chi_{8,c})\bar{\chi}_{8,c} \sum_{p\in\Gamma_{1,1}} (1 - e^{2\pi i p\cdot v}) q^{\frac{1}{2}p_L^2}\bar{q}^{\frac{1}{2}p_R^2}\,.
\end{aligned}
\tag{2.4}
$$

The twisted part of the partition function can be determined by modular transformation since $Z_{(g,1)}(-1/\tau) = Z_{(1,g)}(\tau)$ and $Z_{(1,g)}(\tau+1) = Z_{(g,g)}(\tau)$, the twisted sector contribution is found to be

$$
\begin{aligned}
Z_{tw} &= \frac{1}{2}(Z_{(1,g)} + Z_{(g,g)}) \\
&= \int \frac{d^2\tau}{\tau_2}\tau_2^{-9/2}\frac{1}{|\eta(\tau)|^{16}}(\chi_{8,v} - \chi_{8,c})\bar{\chi}_{8,o} \sum_{p\in\Gamma_{1,1}+v} (1 - e^{2\pi i p\cdot v}) q^{\frac{1}{2}p_L^2}\bar{q}^{\frac{1}{2}p_R^2} - \\
&\quad - \int \frac{d^2\tau}{\tau_2}\tau_2^{-9/2}\frac{1}{|\eta(\tau)|^{16}}(\chi_{8,v} - \chi_{8,c})\bar{\chi}_{8,s} \sum_{p\in\Gamma_{1,1}+v} (1 + e^{2\pi i p\cdot v}) q^{\frac{1}{2}p_L^2}\bar{q}^{\frac{1}{2}p_R^2}\,.
\end{aligned}
\tag{2.5}
$$

The complete partition function is then given by $Z = Z_{untw} + Z_{tw}$. The momenta in the four different sectors are given by

$$
\begin{aligned}
\Lambda_1 = \{p\in\Gamma_{1,1}, (-)^{p.v} = +1\}:&\quad \left(p_L = \frac{2m}{R} + n\frac{R}{2},\ p_R = \frac{2m}{R} - n\frac{R}{2}\right), \\
\Lambda_2 = \{p\in\Gamma_{1,1}, (-)^{p.v} = -1\}:&\quad \left(p_L = \frac{2m+1}{R} + n\frac{R}{2},\ p_R = \frac{2m+1}{R} - n\frac{R}{2}\right), \\
\Lambda_3 = \{p\in\Gamma_{1,1}+v, (-)^{p.v} = +1\}:&\quad \left(p_L = \frac{2m}{R} + \left(n+\frac{1}{2}\right)\frac{R}{2},\right. \\
&\quad \left. p_R = \frac{2m}{R} - \left(n+\frac{1}{2}\right)\frac{R}{2},\right) \\
\Lambda_4 = \{p\in\Gamma_{1,1}+v, (-)^{p.v} = -1\}:&\quad \left(p_L = \frac{2m+1}{R} + \left(n+\frac{1}{2}\right)\frac{R}{2},\right. \\
&\quad \left. p_R = \frac{2m+1}{R} - \left(n+\frac{1}{2}\right)\frac{R}{2}\right).
\end{aligned}
\tag{2.6}
$$

An important feature of this partition function is that at $R = 2$ there are additional massless states coming from the twisted sector (2.5) in $(\chi_{8,v} - \chi_{8,c})\bar{\chi}_{8,o}$. These states are responsible for the enhanced SU(2) gauge symmetry. The closed string spectrum has $(4,0)$ space-time supersymmetry, which is evident from the appearance of $\chi_{8,v} - \chi_{8,c}$ in the partition function Z.

The closed string spectrum can equivalently be described [1] by replacing the standard leftmoving GSO projection $(-1)^{f_L} = +1$ with a modified projection $(-1)^{f_L+p\cdot v} = +1$, where f_L is the leftmoving worldsheet fermion number.

3. Non-BPS D-branes

A very useful tool in the description of D branes is provieded by boundary states [19]–[22]. There are several conditions which are imposed on boundary states in string theories. The boundary state has to preserve (super)-conformal invariance. On the half plane this is equivalent to continuity conditions on the stress tensor and its superpartner, which translate into the following conditions on the boundary states,

$$(L_n - \bar{L}_{-n})|B,\eta\rangle = 0\,, \qquad (G_k + i\eta\bar{G}_{-k})|B,\eta\rangle = 0\,, \tag{3.1}$$

where $\eta = \pm 1$ is related to the spin structure for worldsheet fermions. In most cases the string background has a simple (factorized) dependence on the ghosts and superghosts. Then (super) conformal invariance (3.1) is equivalent to BRST invariance

$$(Q_{BRST} + \bar{Q}_{BRST})|B\rangle = 0\,. \tag{3.2}$$

It is important that the boundary state itself is not a state in the closed string Hilbert space (it has infinite norm), but when expanded in terms of closed string modes only states which appear in the closed string partition function (2.2) appear. Note that the closed string states appearing in the boundary state are 'off shell' because they do not satisfy the $L_0 + \bar{L}_0$ mass shell condition (although (3.1) implies that they satisfy the level matching condition).

An important consistency conditions on boundary states are the Cardy constraints [23] which demand that the cylinder partition function constructed from a boundary state has an open string interpretation. This means that under world sheet duality the cylinder partition function maps into an open string partition function.

The boundary states will be constructed in a lightcone frame for simplicity, the covariant construction of D-brane boundary states is discussed (for example) in [24]. In the NS-NS sector a boundary state (in the lightcone frame) will be of the form

$$\begin{aligned}|B,\eta\rangle_{\rm NSNS} = \mathcal{N}_{\rm NSNS} \exp\left(\sum_n \frac{1}{n}\big(-a^i_{-n}\bar{a}^i_{-n} + a^a_{-n}\bar{a}^a_{-n}\big)\right) \times \\ \times \exp\left(+i\eta\sum_r\big(-b^i_{-r}\bar{b}^i_{-r} + b^a_{-r}\bar{b}^a_{-r}\big)\right)|B,\eta\rangle_0\,. \end{aligned}\tag{3.3}$$

Here the indices i and a run over Neumann and Dirichlet directions respectively. $\mathcal{N}_{\rm NSNS}$ is a normalization factor which depends on the boundary conditions in the compact and noncompact directions. $|B\rangle_0$ denotes the zero mode part of the NS-NS boundary state. If the boundary condition in the compactified direction is Neumann, the boundary state will contain a sum over winding modes. If it is Dirichlet it will contain only a sum over even momentum modes.

$$
\begin{aligned}
|N\rangle_0 &= \sum_n \left| p_L = \frac{nR}{2}, p_R = -\frac{nR}{2} \right\rangle , \\
|D, x\rangle_0 &= \sum_m e^{i2mx/R} \left| p_L = \frac{2m}{R}, p_R = \frac{2m}{R} \right\rangle . \qquad (3.4)
\end{aligned}
$$

The NS-NS boundary state has to be invariant under the right-moving GSO projection $\frac{1}{2}(1-(-1)^{f_R})$. This selects the following combination of the $\eta = \pm 1$ boundary states

$$
|B\rangle_{\mathrm{NSNS}} = |B, +\rangle_{\mathrm{NSNS}} - |B, -\rangle_{\mathrm{NSNS}} . \qquad (3.5)
$$

Note that in the untwisted NSNS sector it is possible to impose either N or D boundary conditions on the compact coordinate. Since all states appearing in (3.4) have even $p \cdot v$ the boundary states also satisfy the modified leftmoving GSO projection.

In the R-R sector a boundary state (in the lightcone frame) will be of the form

$$
\begin{aligned}
|B, \eta\rangle_{\mathrm{RR}} = \mathcal{N}_{\mathrm{RR}} \exp\Bigg(& \sum_n \frac{1}{n}\big(- a^i_{-n}\bar{a}^i_{-n} + a^a_{-n}\bar{a}^a_{-n}\big) + \\
& + i\eta \sum_r \big(- d^i_{-n}\bar{d}^i_{-n} + d^a_{-n}\bar{d}^a_{-n}\big)\Bigg) |\eta\rangle^0_{\mathrm{RR}} . \qquad (3.6)
\end{aligned}
$$

Defining

$$
d^\mu_\pm = \frac{1}{\sqrt{2}}(d^\mu_0 \pm i\bar{d}^\mu_0) . \qquad (3.7)
$$

Where the zero mode piece satisfies

$$
\begin{aligned}
d^i_\eta |\eta\rangle^0_{\mathrm{RR}} &= 0 , \qquad i = \text{Neumann} , \\
d^a_{-\eta} |\eta\rangle^0_{\mathrm{RR}} &= 0 , \qquad a = \text{Dirichlet} . \qquad (3.8)
\end{aligned}
$$

Since $(-1)^{f_R}|B,\eta\rangle = |B - \eta\rangle$, invariance under the rightmoving GSO projection implies that the RR part of the boundary state is of the following form

$$
|B\rangle_{\mathrm{RR}} = |B, +\rangle_{\mathrm{RR}} + |B, -\rangle_{\mathrm{RR}} . \qquad (3.9)
$$

The RR potentials which appear, together with the boundary conditions in the compact directions are determined by the closed string partition function. Only states which appear in the closed string partition function Z (defined in (2.4) and (2.5)) can be used to construct the boundary states.

In the partition function there are two sectors with RR-fields, one coming from the untwisted sector (2.4), $\chi_{8,c}\bar{\chi}_{8,c}$ and one coming from $\chi_{8,c}\bar{\chi}_{8,s}$ in (2.5).

Since the momenta associated with $\chi_{8,c}\bar{\chi}_{8,c}$ sector lie in Λ_2, Neumann boundary conditions in the compact directions $p_L = -p_R$ are inconsistent. Only Dirichlet boundary conditions $p_L = p_R$ will be consistent.

The generalized leftmoving GSO projection $(-1)^{f_L + p\cdot v} = +1$ acting on the boundary state selects RR field strengths with p odd (in agreement with the field content in $\chi_{8,c}\bar{\chi}_{8,c}$). The compact momentum part of the boundary state is

$$|B\rangle^D_{0,RR} = \sum_m e^{i(2m+1)x/R} \left| p_L = \frac{2m+1}{R}, p_R = \frac{2m+1}{R} \right\rangle \otimes |p = 2k-1\rangle_0 \,. \tag{3.10}$$

The boundary state with Dirichlet boundary conditions in the compact directions has a simple interpretation as a superposition of a $D(2k-1)$ brane at x and a $\bar{D}(2k-1)$ brane at $x+\pi R$, since the action of $(-1)^{F_L}\sigma_{1/2}$ reverses the RR charge and translates the position of the brane by πR. The massless RR part of the boundary state cancel between the brane and the antibrane and only massive RR states nonvanishing momenta contribute.

The second sector of RR fields lies in the twisted sector (2.5) $\chi_{8,c}\bar{\chi}_{8,s}$. Note that because of the momenta which appear in this sector Λ_3, Dirichlet boundary conditions $p_L = p_R$ are inconsistent. Only Neumann boundary conditions $p_L = -p_R$ will be consistent. Since the momenta in Λ_3 have even $p\cdot v$, the generalized leftmoving GSO projection selects RR field strengths with p even (in agreement with the field content in $\chi_{8,c}\bar{\chi}_{8,s}$). The compact momentum part of the boundary state is

$$|B\rangle^N_{0,RR} = \sum_n \left| p_L = \left(n + \frac{1}{2}\right)\frac{R}{2}, p_R = -\left(n+\frac{1}{2}\right)\frac{R}{2} \right\rangle \otimes |p = 2k\rangle_0 \,. \tag{3.11}$$

Again this boundary state can be interpreted as a superposition of a $D(2k)$ and $\bar{D}(2k)$ wrapped on the circle with a nontrivial Wilson line.

4. Open string spectrum

The boundary states can be used to construct a cylinder diagram which has the interpretation of a closed string tree level exchange diagram. World sheet duality turns the cylinder into an annulus, which has the interpretation of a one loop open string diagram and hence determines open string spectrum.

The contribution to the cylinder amplitude coming from the NS-NS sector will depend on whether we have D or N boundary conditions on the circle.

$$\begin{aligned} Z^N_{\rm NS} &= {}_{\rm NS}\langle B, N| \int dl e^{-(\pi l(L_0 + \bar{L}_0 - 1))} |B, N\rangle_{\rm NS} \\ &= 2(\mathcal{N}^N_{\rm NSNS})^2 \int dl\; l^{(p-8)/2} \frac{1}{\eta^{12}(q)} (\theta_3^4(q) - \theta_4^3(q)) \sum_n e^{-\pi l n^2 R^2/4} \end{aligned} \tag{4.1}$$

$$\begin{aligned} Z^D_{\rm NS} &= {}_{\rm NS}\langle B,D|\int dl e^{-(\pi l(L_0+\bar L_0-1))}|B,D\rangle_{\rm NS} \\ &= 2(\mathcal{N}^D_{\rm NSNS})^2\int dl\; l^{(p-8)/2}\frac{1}{\eta^{12}(q)}(\theta_3^4(q)-\theta_4^3(q))\sum_m e^{-\pi l 4m^2/R^2}\,. \end{aligned} \tag{4.2}$$

Where $q=e^{-2\pi l}$ and $p+1$ denotes the number of noncompact Neumann directions. The contribution for Neumann boundary condition from the RR part is

$$\begin{aligned} Z^N_{\rm RR} &= {}_{\rm RR}\langle B,D|\int e^{-(\pi t(L_0+\bar L_0))}|B,D\rangle_{\rm RR} \\ &= \frac{1}{8}(\mathcal{N}^N_{\rm RR})^2\int dl\; l^{(p-8)/2}\frac{1}{\eta^{12}(q)}\theta_2^4(q)\sum_m e^{-\pi l(m+1/2)^2R^2/4}\,. \end{aligned} \tag{4.3}$$

The contribution to the cylinder amplitude coming from the RR sector for Dirichlet boundary condition on the circle are given by

$$\begin{aligned} Z^D_{\rm RR} &= {}_{\rm RR}\langle B,D|\int dt e^{-(\pi l(L_0+\bar L_0))}|B,D\rangle_{\rm RR} \\ &= \frac{1}{8}(\mathcal{N}^D_{\rm RR})^2\int dl\; l^{(p-8)/2}\frac{1}{\eta^{12}(q)}\theta_2^4(q)\sum_m e^{-\pi l4(m+1/2)^2/R^2} \end{aligned} \tag{4.4}$$

There are now two possible boundary states which are associated with having either Neumann or Dirichlet boundary conditions in the compact direction.

$$Z^D = Z^D_{\rm NS}+Z^D_R\,,\qquad Z^N = Z^N_{\rm NS}+Z^N_R\,. \tag{4.5}$$

The open string partition functions are obtained by modular transformation $t=1/l$. For the Dirichlet boundary conditions one finds

$$\begin{aligned} \tilde Z^D &= R\int\frac{dt}{t}t^{-(p+1)/2}\frac{1}{\eta^{12}(w)}\Big((\mathcal{N}^D_{\rm NSNS})^2(\theta_3^4(w)-\theta_2^4(w))\sum_m e^{-\pi tR^2m^2/4}+ \\ &\qquad\qquad +\frac{1}{16}(\mathcal{N}^D_{\rm RR})^2\theta_4^4(w)\sum_m(-1)^m e^{-\pi tR^2m^2/4}\Big) \qquad (4.6) \\ &= \frac{V_{p+1}}{(4\pi)^{p+1}}\int\frac{dt}{t}t^{-(p+1)/2}\frac{1}{\eta^{12}(w)}\Big(\chi_{8,v}\sum_m e^{-\pi tR^2m^2}+\chi_{8,o}\sum_m e^{-\pi tR^2(m+1/2)^2}- \\ &\qquad\qquad -\chi_{8,c}\sum_m e^{-\pi tR^2m^2}-\chi_{8,s}\sum_m e^{-\pi tR^2(2m+1)^2/4}\Big). \end{aligned}$$

Where $w=e^{-2\pi t}$ and the normalization factors are determined by demanding that (4.6) has the interpretation of an open string partition function $\tilde Z=\mathrm{tr}(w^H\mathcal{P})$ where the generalized GSO projection is given by $\mathcal{P}=\frac{1}{2}(1+(-1)^{f+m^2})$. Here odd winding numbers correspond to strings winding around half the circle of radius R, i.e. open strings starting at a brane at $x=x_0$ and ending at an antibrane at $x=x_0+\pi R$. The normalization factors are the determined

$$(\mathcal{N}^D_{\rm NSNS})^2 = R\frac{V_{p+1}}{(4\pi)^{p+1}}\,,\qquad (\mathcal{N}^N_{\rm RR})^2 = -16\;R\,\frac{V_{p+1}}{(4\pi)^{p+1}}\,. \tag{4.7}$$

For the Neumann boundary conditions one finds

$$\begin{aligned}\tilde{Z}^N &= \frac{4}{R}\int\frac{dt}{t}t^{-(p+1)/2}\frac{1}{\eta^{12}(w)}\Bigg((\mathcal{N}^N_{\rm NSNS})^2(\theta_3^4(w)-\theta_2^4(w))\sum_m e^{-\pi t4m^2/R^2}+\\ &\qquad\qquad + \frac{1}{16}(\mathcal{N}^N_{\rm RR})^2\theta_4^4(w)\sum_m(-1)^m e^{-\pi t4m^2/R^2}\Bigg) \qquad (4.8)\\ &= \frac{V_{p+1}}{(4\pi)^{p+1}}\int\frac{dt}{t}t^{-(p+1)/2}\frac{1}{\eta^{12}(w)}\Bigg(\chi_{8,v}\sum_m e^{-\pi t16m^2/R^2}+\chi_{8,o}\sum_m e^{-\pi t16(m+1/2)^2/R^2}-\\ &\qquad\qquad - \chi_{8,c}\sum_m e^{-\pi t16m^2/R^2}-\chi_{8,s}\sum_m e^{-\pi t4(2m+1)^2/R^2}\Bigg).\end{aligned}$$

The open string amplitude is consistent with the following generalized GSO projection $\mathcal{P}=\frac{1}{2}(1-(-1)^{f+m^2})$. This determines

$$(\mathcal{N}^N_{\rm NSNS})^2=\frac{4}{R}\frac{V_{p+1}}{(4\pi)^{p+1}}\,,\qquad (\mathcal{N}^N_{\rm RR})^2=-\frac{64}{R}\frac{V_{p+1}}{(4\pi)^{p+1}}\,. \qquad (4.9)$$

Note that in certain ranges of the radius R there is a tachyon in the spectrum coming from the χ_o sector. However there is no tachyon in $\tilde{Z}_D$ for $R>2$ and there is no tachyon in $\tilde{Z}_N$ for $R<2$. Hence there is always a non-BPS brane which is does not have a tachyon in the open string spectrum and is therefore stable. Note also that the critical radius $R=2$ is the radius where an enhanced gauge symmetry in the bulk appears. The two partition functions $\tilde{Z}^D$ (4.6) and $\tilde{Z}^N$ (4.8) are equal and they can be expressed as at that radius as

$$Z|_{R=2}=\frac{V_{p+1}}{(4\pi)^{p+1}}\int\frac{dt}{t}t^{-(p+1)/2}\frac{1}{\eta^{12}(w)}\Big(\theta_3^5(w)-\theta_4^5(w)-\theta_2^4(w)\theta_3(w)\Big). \qquad (4.10)$$

The contribution to the partition function coming from the massless sector vanishes, which indicates a degeneracy between the number of bosons and fermions in the massless spectrum. However neither is there such a degeneracy at higher levels nor are the massless interactions supersymmetric.

5. Free fermion construction

A compactification of type-II strings from ten to d dimensions uses an internal SCFT with central charge $c=3/2(10-d)$ The free fermionic construction [3, 4] uses an internal conformal system is made of $3(10-d)$ free left and rightmoving fermions. Different models come from the choice of spin structures (or equivalently generalized GSO) projections, consistent with modular invariance, spin statistics and factorization. Since the left and rightmoving fermions can have different generalized GSO projections many interesting models with various numbers of left and rightmoving spactime supersymmetries and gauge groups are possible to construct [5, 25].

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The simplest case is given by a compactification to nine dimension using three real fermions. Indeed the asymmetric orbifold discussed in section 4 at radius $R = 2$ can be represented using free fermions. The internal fermion ψ_9 and the internal boson X_9 are replaced by three real fermions $\lambda_1, \lambda_2, \lambda_3$, via $\psi_9 = \lambda_1, \partial X_9 = \lambda_2\lambda_3$. Hence the worldsheet fields in the nine dimensional theory are given by

$$\text{left: } \psi^\mu, \lambda_1, \lambda_2, \lambda_3 \qquad \text{right: } \bar{\psi}^\mu, \bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3\,, \qquad \mu = 0, \ldots, 8\,. \tag{5.1}$$

The worldsheet supercurrents are given by

$$T_F = i\left(\sum_{\mu=0}^{8} \partial_z X^\mu \psi_\mu + \lambda_1\lambda_2\lambda_3\right), \qquad \bar{T}_F = i\left(\sum_{\mu=0}^{8} \partial_{\bar{z}} X^\mu \bar{\psi}_\mu + \bar{\lambda}_1\bar{\lambda}_2\bar{\lambda}_3\right). \tag{5.2}$$

In the free fermionic construction of the asymmetric orbifold compactification [4] one selects two sets of fermions $S = \{\psi^\mu, \lambda_1\}$ and $F = \{\lambda_2, \lambda_3, \bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3, \bar{\psi}^\mu\}$. There are four sectors $0, F, S, F/S$ and two GSO projections $(-1)^F = +1$ and $(-1)^S = +1$. The closed string partition function is then given by

$$Z = \int \frac{d^2\tau}{\tau_2^2}\tau_2^{p/2}\frac{1}{|\eta(\tau)|^{16}}(\chi_{8,v} - \chi_{8,c})\frac{1}{\bar{\eta}^4}\left(\bar{\theta}_3^4\,|\theta_3|^2 - \bar{\theta}_4^4\,|\theta_4|^2 - \bar{\theta}_2^4\,|\theta_2|^2\right). \tag{5.3}$$

In the NS-NS sector the boundary conditions can be encoded as

$$\begin{aligned}
(\psi_r^i + i\eta_\psi\bar{\psi}_{-r}^i)|B\rangle &= 0\,, \qquad i = \text{Neumann}\,,\\
(\psi_r^a - i\eta_\psi\bar{\psi}_{-r}^a)|B\rangle &= 0\,, \qquad a = \text{Dirichlet}\,,\\
(\lambda_r^i + iM_j^i\bar{\lambda}_{-r}^j)|B\rangle &= 0\,.
\end{aligned} \tag{5.4}$$

Where $\eta_\psi = \pm 1$ and M is an $O(3)$ matrix. By a suitable choice of basis M can be expressed as $M = diag(\eta_1, \eta_2, \eta_3)$ with $\eta_i = \pm 1$. These choices determine the boundary state $|B, \eta_\psi, \eta_1, \eta_2, \eta_3\rangle$. The condition of worldsheet superconformal invariance (3.1) with T_F defined by (5.2) implies the following condition

$$\det(M) = \eta_1\eta_2\eta_3 = \eta_\psi\,. \tag{5.5}$$

The boundary state for the internal fermions λ_i can be represented as a coherent state imposing the conditions (5.4). The action of the right-moving GSO projection $(-1)^S$ acts as $(-1)^{f_R}$ (where f_R is the worldsheet fermion number associated with ψ_μ, λ_1) on such a boundary state

$$(-1)^S|B, \eta_\psi, \eta_1, \eta_2, \eta_3\rangle_{\rm NS} = -|B, -\eta_\psi, -\eta_1, \eta_2, \eta_3\rangle_{\rm NS}\,. \tag{5.6}$$

The other GSO projection in this model is $(-1)^F$ and involves both λ_2, λ_3 and $\bar{\lambda}_2, \bar{\lambda}_3$ and does therefore not change the sign of η_2, η_3 and acts effectively as $(-1)^{f_L}$. One possible boundary state in the NS-NS sector is given by[1]

$$|B\rangle_{\rm NSNS}^{N} = |B, 1, 1, 1, 1\rangle_{\rm NS} - |B, -1, -1, 1, 1\rangle_{\rm NS}\,. \tag{5.7}$$

[1]The normalizations of the boundary states are determined in the same way as in section 4, and given by setting $R = 2$.

Note that this choice of signs is equivalent to imposing Neumann boundary conditions on the original compact boson X^9. A different choice is

$$|B\rangle^{D}_{\mathrm{NSNS}} = |B,1,-1,-1,1\rangle - |B,-1,1,-1,1\rangle\,, \tag{5.8}$$

which corresponds to Dirichlet boundary condition on the boson X^9. Note that the two boundary conditions are related by an SO(3) rotation of the boundary conditions which acts on the matrix M as

$$M \to \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix} M\,, \tag{5.9}$$

which send $\eta_{1,2} \to -\eta_{1,2}, \eta_3 \to \eta_3$ and hence maps (5.7) into (5.8). For both boundary states the NS-NS part of the cylinder partition function is given by

$$Z_{\mathrm{NS}} = \langle B|\Delta|B\rangle_{\mathrm{NSNS}} = \int dl\; l^{(8-p)/2}\frac{1}{\eta^{12}(q)}(\theta_3^4(q) - \theta_4^4(q))\theta_3(q)\,. \tag{5.10}$$

In the RR sector the boundary state can be buildt as a coherent state acting on a zero mode part. The zero mode part is constructed by creation and annihilation operators

$$\psi^\mu_\eta = \psi^\mu_0 + i\eta\bar\psi^\mu_0, \lambda^i_\eta = \lambda^i_0 + i\eta_i\bar\lambda^i_0\,, \qquad i = 1,2,3\,. \tag{5.11}$$

The zero mode piece will then be (in the lightcone gauge) a bispinor of SO(10). Hence these states will be massive. The zero mode part $|B,\eta_\psi,\eta_1,\eta_2\eta_3\rangle^{\mathrm{RR}}$ will satisfy

$$\begin{aligned} \psi^\mu_{\eta_\psi}|B\rangle &= 0\,, \qquad \mu = 1,\ldots,7 \\ \lambda^i_{\eta_i}|B\rangle &= 0\,, \qquad i = 1,2,3\,. \end{aligned} \tag{5.12}$$

The GSO projection $(-1)^S$ implies that only the combination

$$|B^{\mathrm{RR}}\rangle = |B,\eta_\psi,\eta_1,\eta_2,\eta_3\rangle^{\mathrm{RR}} + |B,-\eta_\psi,-\eta_1,\eta_2,\eta_3\rangle^{\mathrm{RR}} \tag{5.13}$$

is physical. The projection $(-1)^F$ determines which bispinors of SO(10) appear in (5.13), depending on the choice of Dirichlet and Neumann boundary conditions in the 9-th direction.

The RR contribution to the cylinder partition function (with the correct normalization of the boundary states) will be of the form θ_2^5/η^{12} and the complete cylinder partition function is given by

$$Z_{cyl} = \frac{V_{p+1}}{(4\pi)^{p+1}}\int dl l^{(8-p)/2}\frac{1}{\eta^{12}(q)}\Big((\theta_3^4(q) - \theta_4^4(q))\theta_3(q) - \theta_2^5(q)\Big)\,. \tag{5.14}$$

After a modular transformation to the open string channel this partition function maps into (4.10). Hence the free fermions provide an equivalent representation of the enhanced symmetry point $R = 2$ of the asymmetric orbifold discussed in section 4.

6. Gauge symmetry and stability

In the asymmetric orbifold discussed above there is an enhancement of gauge symmetry in the bulk at $R = 2$. A vertex operator for the SU(2) gauge bosons will be of the form (in the zero ghost picture)

$$V(\zeta) = \zeta^a_\mu \int d^2z(\partial X^\mu + ik_\nu \psi^\nu \psi^\mu)\bar{J}^a e^{ikX} . \tag{6.1}$$

Where the currents $\bar{J}^a$ form a $\mathrm{SU}(2)_2$ current algebra. In the free fermionic construction they are given by $\bar{J}^a = \frac{i}{2}\epsilon^{abc}\bar{\lambda}_b\bar{\lambda}_c$. On shell gauge invariance manifests itself in closed string amplitudes in the following way: replacing the wave function ζ^μ_a by $k_\mu \epsilon^a$ turns vertex operator into a total derivative, which implies that the corresponding closed string amplitudes $\langle V(k_\mu\epsilon^a)V_1V_2\cdots V_n\rangle$ vanish. In the presence of a worldsheet boundary/D-brane, the total derivative can pick up a boundary term. In the case at hand such a boundary term acts like an infinitesimal SU(2) rotation of the boundary state.

$$V(k_\mu\epsilon^a) = \epsilon^a \int d^2z\partial(\bar{J}^a e^{ikX}) = \epsilon^a \oint \bar{J}^a e^{ikX} . \tag{6.2}$$

This means that a gauge transformation in the bulk induces a shift shift in the scalars living on the brane. This is reminiscent of [26], where it was shown that in the case of the bosonic string with enhanced gauge symmetry $G\times G$ the massless open string scalars act like Goldstone bosons and effectively break the bulk gauge symmetry to $(G \times G)/G$. If the brane does not fill space-time the Goldstone bosons are localized on the worldvolume of a brane of lower dimensions.

Defining the zero mode of the SU(2) current, $J^a_0 = \frac{i}{2}\epsilon^{abc}\sum_n \bar{\lambda}^b_{-n}\lambda^c_n$, a finite gauge transformation in the bulk is equivalent to a constant condensate on the boundary $|B,\epsilon\rangle = \exp(i\epsilon_a J^a_0)|B\rangle$. The constant condensate modifies the boundary conditions on the λ_i

$$\lambda^i_r|B,\epsilon\rangle = M^i_j e^{i\epsilon_a J^a_0}\bar{\lambda}^j_{-r}e^{-i\epsilon_a J^a_0}|B,\epsilon\rangle = M^i_j R(\epsilon)^j_k \bar{\lambda}^k_{-r}|B,\epsilon\rangle. \tag{6.3}$$

Hence a finite boundary condensate parameterized by ϵ^a is equivalent to modified boundary conditions given by $M' = MR$ where R is the SO(3) rotation matrix associated with the adjoint action of $g = \exp(i\epsilon_a J^a_0)$. Hence the rotation matrix $R = \exp(\epsilon^a N^a)$ where the antisymmetric matrix $(N^a)_{ij} = \epsilon_{aij}$. This implies that the Neumann and Dirichlet boundary conditions (5.7) and (5.8) are related by a marginal deformation just like in [27]. This marginal deformation of the boundary state is equivalent to a gauge transformation of the bulk theory.

7. S-duality and type-$\tilde{\mathrm{I}}$ theory

The nine dimensional theory which is obtained by the asymmetric orbifold $(-)^{F_L}\sigma_{1/2}$ of type IIB has sixteen real supersymmetries. At generic values of the compactification radius the gauge group is $\mathrm{U}(1)^2$ and comes from the Kaluza Klein reduction of the graviton and the antisymmetric tensor in ten dimensions. This theory belongs to the $\mathcal{M}_{1,1}$ component of the moduli space of consistent theories with 16 supercharges in 9 dimensions [9, 28, 29, 30]. This class of theories has another description as an orientifold which is called type-$\tilde{\mathrm{I}}$ theory and can be obtained from type IIB compactified on a circle by gauging the Z_2 symmetry $\Omega\sigma_{1/2}$ [31, 32], where Ω denotes worldsheet parity tranformation. There is also a T-dual description of type-$\tilde{\mathrm{I}}$ theory (called $\tilde{\mathrm{IA}}$) which has two orientifold 8-planes of opposite charge sitting at the two endpoints of an interval. An analysis of BPS and non BPS branes in this theory using K-theory was given in [33].

The asymmetric $(-1)^{F_L}\sigma_{1/2}$ orbifold and type $\tilde{\mathrm{I}}$ are related by an S duality [32, 34] as follows. Since the two perturbative symmetries of type IIB $(-1)^{F_L}$ and Ω are mapped into each other via $(-1)^F_L = S\Omega S^{-1}$ [35], the orbifolds involving half period shifts are dual because the adiabatic argument of Vafa and Witten [36] applies.

Note that under an S duality of the parent IIB theory the fundamental string and NS 5-brane are mapped to the D string and D 5-brane respectively. This is in accord with the fact that in type-$\tilde{\mathrm{I}}$ theory has the same BPS D branes a type-I theory [33], whereas in the $(-)^{F_L}\sigma_{1/2}$ all 1/2-BPS objects are perturbative string states. Some aspects of these theories and their relation were discussed in [34, 37] and the relation of BPS saturated amplitudes one loop amplitudes was discussed in [38, 39].

An interesting question is how the enhanced SU(2) gauge symmetry of the $(-)^{F_L}\sigma_{1/2}$ orbifold at $R = 2$ manifests itself in the dual type-$\tilde{\mathrm{I}}$ theory (or in the T-dual type-$\tilde{\mathrm{I}}$A theory). The masses of the states which become massless at the critical radius $R = 2$ in the partition function (2.5) are (in the string frame)

$$m^2 = \left(\frac{R}{4} - \frac{1}{R}\right)^2 . \tag{7.1}$$

Note that these states fall into short (256 dimensional) supermultiplets. After an S-duality transformation this mass formula for the states in type $\tilde{\mathrm{I}}$ has the form (in the dual string frame)

$$\tilde{m}^2 = \left(e^{-\tilde{\phi}}\frac{\tilde{R}}{4} - \frac{1}{\tilde{R}}\right)^2 . \tag{7.2}$$

This indicates that the enhancement of gauge symmetry in the type-$\tilde{\mathrm{I}}$ theory is caused by a D string becoming massless. For this interpretation to be valid one has to continue (7.1) to strong coupling, which is justified for BPS states.

After a T-duality on the circle the corresponding mass formula for the type $\tilde{\mathrm{I}}$A theory is given by

$$\tilde{m}^2 = \left(e^{-\tilde{\phi}}\frac{1}{4} - \tilde{R}\right)^2 . \tag{7.3}$$

Just like in the discussion of enhanced gauge symmetry in type-I' theory [37] the enhancement of the gauge symmetry should be visible small coupling and large radius in the type $\tilde{\text{I}}$A theory, where a perturbative analysis should suffice. Type $\tilde{\text{I}}$A theory is an orientifold with a O_8^+ and O_8^- orientifold plane at the end of an interval. Since the RR charges are canceled between the orientifold planes, no D8 branes have to be introduced (in contrast to type I'). Although the properties of O_8^- orientifold planes in supergravity are not well understood, it is quite likely that their properties at least what the supergravity is concerned (i.e. not taking open strings into account) are well approximated by 16 D8-branes on top of an O_8^+. Following [37] the string coupling will diverge near the orientifold planes making the SU(2) gauge enhancement an intrinsically nonperturbative phenomenon in $\tilde{\text{I}}$A theory. It would be very interesting to see whether this enhancement of gauge symmetry can be analyzed in the $\tilde{\text{I}}$A theory using the methods developed in [40], where presumably it comes from bound states of D0 branes near an orientifold plane.

A M-theory realization of these nine dimensional theories is given by a compactification of M-theory on a Klein bottle, which is not purely geometrical since the Klein bottle is realized as a freely acting orbifold of the torus, where the geometric action is accompanied with a sign reversal of the three form potential $C \to -C$ [32].

8. Discussion

In this paper we have discussed the construction of non BPS D-branes in a very simply nine dimensional symmetric orbifold, generated by $(-1)^{F_L}\sigma_{1/2}$. The interesting features of these D-branes are that although they do not carry any RR charge, the boundary state contains a (massive) RR part which is important for consistency and stability. There are two kinds of D branes, one with Dirichlet boundary conditions in the compact direction which is stable for $R > 2$ and one with Neumann boundary conditions which is stable for $R < 2$. At the critical point $R = 2$, when these D-branes become unstable, there is an enhanced gauge symmetry in the bulk. The deformation of the boundary conditions from Neumann to Dirichlet can then be interpreted as a global gauge transformation in the bulk. Since the D-branes are non BPS they cannot simply be continued to strong coupling.

Since they are stable, euclidean D-branes will however correspond to D-instantons, which can give dominant nonperturbative contributions to certain processes. Such non-BPS D-instantons will have sixteen fermionic zero modes and possibly contribute to R^4 terms. It would be interesting to investigate the effects of these non-BPS instantons further.

The asymmetric orbifold which was considered in this paper is a very 'mild' one since it only involves a non geometric action $(-1)^{F_L}$. It would be very interesting to try to generalize this to more complicated asymmetric orbifolds. It might be that the fact that some non geometric asymmetric orbifolds have free fermionic realizations at special points in their moduli space will be very useful, since the boundary states are very easy to construct for the free fermions and the consistency condition, i.e. the imposition of the generalized GSO projections in addition to the Cardy constraints could be more managable.

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